Microwave radiometric mapping of broken cumulus cloudfields from space: numerical simulations

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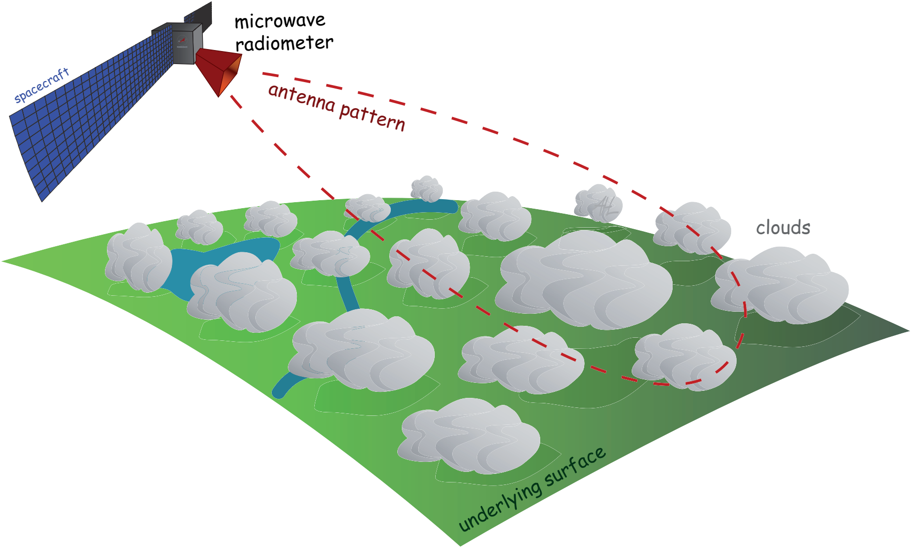


Figure 1: Radiometric sounding of a cloudy atmosphere from space

Brightness temperature $T_b^\nu(\theta)$ (K) spectrum is the main characteristic of microwave radiation.

$$
\begin{aligned}
T_b^\nu(\theta) = {} & T_s \cdot \varkappa_\nu(\theta) \cdot \exp\left(-\tau_\nu \sec\theta\right) + \\
& + \int_0^\infty T(h)\gamma_\nu(h) \sec\theta \cdot \exp\left(-\int_h^\infty \gamma_\nu(z) \sec\theta \, dz\right) dh + \\
& + R_\nu(\theta) \cdot \exp\left(-\tau_\nu \sec\theta\right) \cdot \\
& \cdot \int_0^\infty T(h)\gamma_\nu(h) \sec\theta \cdot \exp\left(-\int_0^h \gamma_\nu(z) \sec\theta \, dz\right) dh + \\
& + R(\theta) \cdot T_C(\phi, \theta) \cdot \exp\left(-2\tau_\nu \sec\theta\right),
\end{aligned}
\tag{1}
$$

$$
\tau_\nu = \int_0^\infty \gamma_\nu(h) \, dh.
$$

Attenuation coefficients (dB/km)

$$\gamma_\nu(h) = \gamma_{O2}(\nu, h) + \gamma_{H2O}(\nu, h) + \gamma_w(\nu, h) + \ldots, \qquad (2)$$

where

$$\gamma_{O2}(\nu, h) = \gamma_{O2}\left(\nu, \mathrm{T}(h), \mathrm{P}(h)\right) \quad \text{(Rec. ITU-R P.676)},$$
$$\gamma_{H2O}(\nu, h) = \gamma_{H2O}\left(\nu, \mathrm{T}(h), \mathrm{P}(h), \rho(h)\right) \quad \text{(Rec. ITU-R P.676)},$$
$$\gamma_w(\nu, h) \approx \frac{60\pi \cdot \nu}{c} \mathrm{Im}\left(\frac{\varepsilon - 1}{\varepsilon + 2}\right) \cdot \mathrm{w}(h) \ \text{ or Rec. ITU-R P.840.}$$

Here ν is radiation frequency; $\mathrm{T}(h)$, $\mathrm{P}(h)$ and $\rho(h)$ are thermodynamic temperature (°C), atmospheric pressure (hPa) and absolute humidity (g/m^3) altitude profiles; $\varepsilon = \varepsilon(\nu, T(h))$ is complex dielectric permittivity of zero-salinity water, and $w(h)$ is altitude profile of liquid water.

Attenuation coefficients (dB/km)

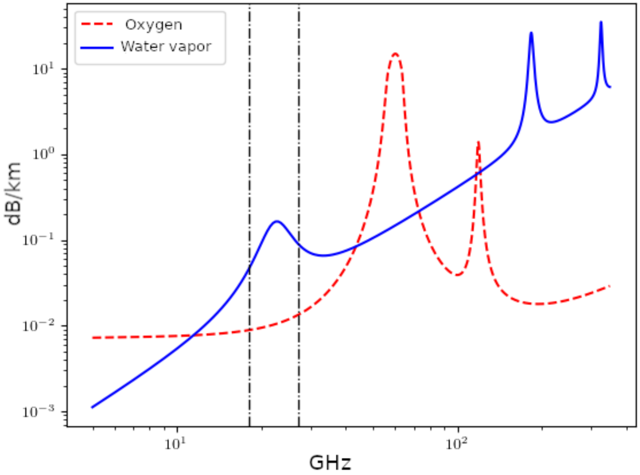


Figure 2: Near-ground frequency spectrum of attenuation coefficients (standard atmosphere, $T_0 = 15°$C, $P_0 = 1013$ hPa, $\rho_0 = 7.5$ g/m^3)

Smooth water surface reflectance

through Fresnel coefficients (v – vertical, h – horizontal polarizations)

$$R_{\mathrm{v,h}}(\theta) = R_{\mathrm{v,h}}(90^\circ - \psi) = \left|M_{\mathrm{v,h}}(\psi)\right|^2, \tag{3}$$

$$M_{\mathrm{h}}(\psi) = \frac{\sin\psi - \left(\varepsilon - \cos^2\psi\right)^{0.5}}{\sin\psi + \left(\varepsilon - \cos^2\psi\right)^{0.5}},$$

$$M_{\mathrm{v}}(\psi) = \frac{\varepsilon\sin\psi - \left(\varepsilon - \cos^2\psi\right)^{0.5}}{\varepsilon\sin\psi + \left(\varepsilon - \cos^2\psi\right)^{0.5}}.$$

In case of $\theta = 0$ deg. (zenith angle)

$$M_{\mathrm{h}} = M_{\mathrm{v}} = \left(\varepsilon^{0.5} - 1\right) \cdot \left(\varepsilon^{0.5} + 1\right)^{-1}. \tag{4}$$

Under conditions of thermodynamic equilibrium

$$\varkappa_{\mathrm{v,h}}(\theta) = 1 - R_{\mathrm{v,h}}(\theta). \tag{5}$$

Complex dielectric permittivity of water

Debye model

$$\varepsilon = \left(\varepsilon_O + \frac{\varepsilon_S - \varepsilon_O}{1 + (\lambda_S/\lambda)^2}\right) - i \cdot \frac{\lambda_S}{\lambda} \cdot \frac{\varepsilon_S - \varepsilon_O}{1 + (\lambda_S/\lambda)^2}. \tag{6}$$

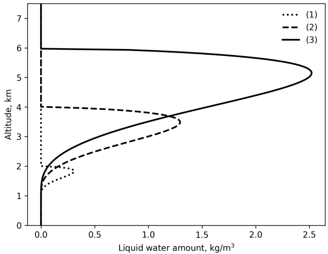
Experimental approximations

$$\begin{aligned} &\varepsilon_O = 5.5 \\ &\varepsilon_S = 88.2 - 0.40885 \cdot T + 0.00081 \cdot T^2, \\ &\lambda_S = 1.8735 - 0.0273 \cdot T + 0.00014 \cdot T^2 + 1.662 \cdot \exp\left(-0.0634 \cdot T\right) \end{aligned} \tag{7}$$

Additional amendments must be made in case of non-zero salinity of water.

Liquid water altitude distribution (cumuli)

a)



b)

Figure 3: a) Anvil-like cloud (cumulus congestus); b) Model altitude distributions of liquid water for cloud layer heights of (1) $H = 1$ km, (2) $H = 3$ km, (3) $H = 5$ km. The cloud base altitude is 1 km

Liquid water altitude distribution (cumuli)

The liquid water profile inside a cumulus cloud can be approximated as follows (Mazin's model)

$$\mathrm{w}(\xi) = \frac{W}{H} \cdot \frac{\Gamma(2+\mu_0+\psi_0)}{\Gamma(1+\mu_0)\Gamma(1+\psi_0)} \xi^{\mu_0} (1-\xi)^{\psi_0}, \qquad (8)$$

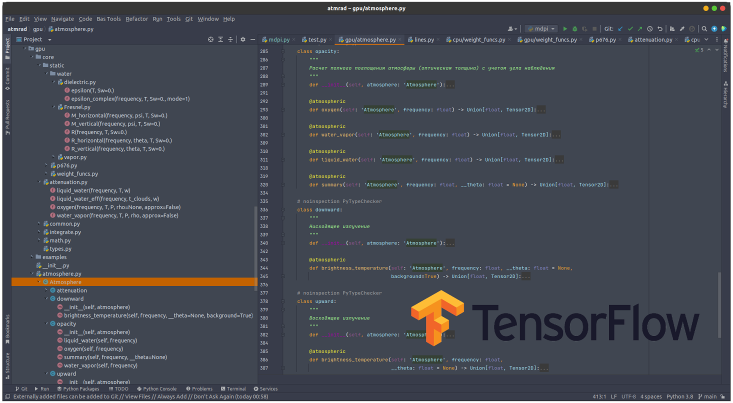
where $\xi = h/H$ is the reduced height; H is cloud power (km); W is integral liquid water content or LWC (kg/m^2); $\mathrm{w}(\xi)$ represents altitude profile of liquid water inside the cloud (kg/m^3); μ_0 and ψ_0 are dimensionless parameters. According to [1], the average values of these parameters are $\mu_0 = 3.27$, $\psi_0 = 0.67$.

Table 1.

Cloud species	W, kg/m^2	T^*_{cl}, °C	H, km
Cumulus humilis	0.15	2.9	1.1
Cumulus mediocris	0.52	-2.0	2.0
Cumulus congerstus	4.73	-14.1	4.5

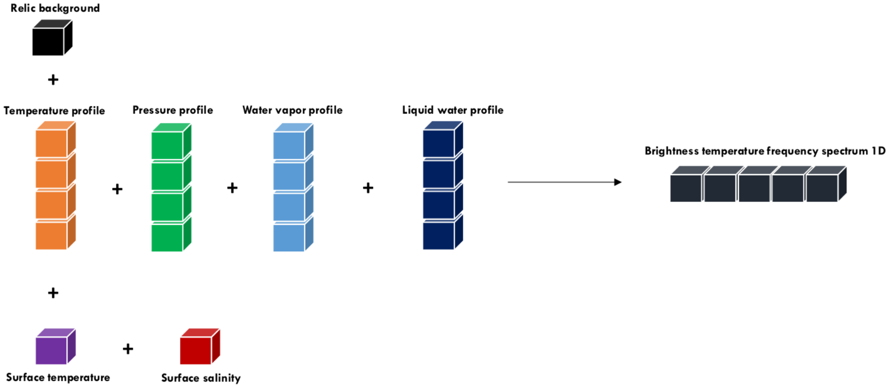
The integral liquid water content W on cloud power H experimental dependence in case of cumuli can be approximated, e.g. as

$$W = 0.132574 \cdot H^{2.30215}. \tag{9}$$



Link to project: https://github.com/dobribobri/atmrad

Data flow [option 1]



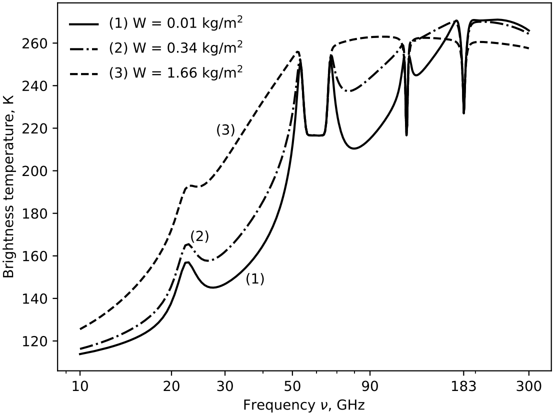


Figure 4: Zenith-outgoing radiation brightness temperature spectra (10-300 GHz) of "smooth water surface – standard atmosphere" system with flat cloud layers of various LWC added. The layer height is approximated according to (9). Mazin's model is utilized for liquid water profile calculation, see (8). Cloud base height is 1.5 km

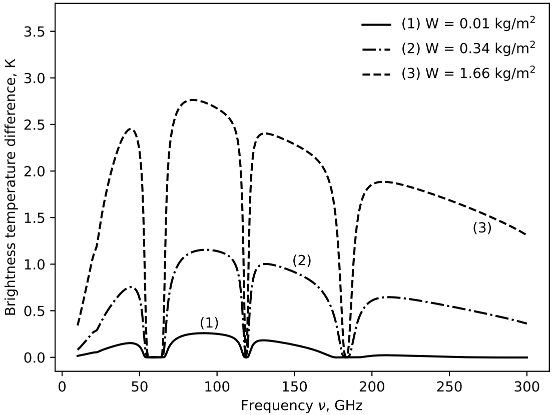


Figure 5: How the difference from brightness temperature spectra given in the previous figure would be, if the liquid water amount does not change with altitude (unlike Mazin's law).

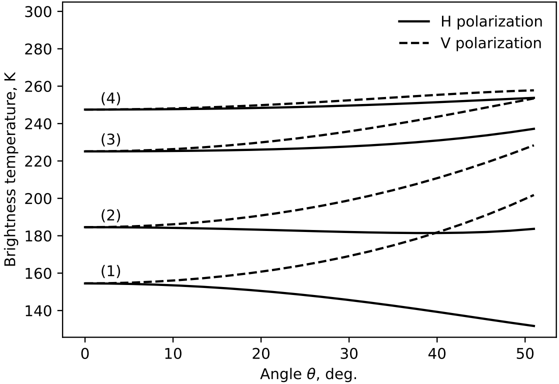
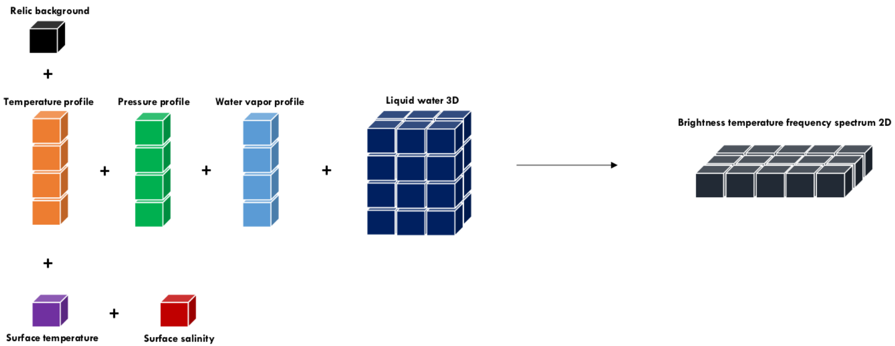


Figure 6: The dependence of "smooth water surface – atmosphere" system outgoing radiation brightness temperature at 36 GHz on changing the observation angle under conditions of (1) clear sky, (2) 0.5 kg/m^2, (3) 1.5 kg/m^2, and (4) 3 kg/m^2 LWC (flat cloud layer). Horizontal (H) and vertical (V) polarizations

Data flow [option 2]



Planck's model for cloudfields

Based on the results of processing an extensive database of stereoscopic survey at Florida, USA, Planck (1969) proposed the following distribution function for cumuli

$$n(D) = K \cdot e^{-\alpha D},\ d \leq D \leq D_m. \tag{10}$$

Here D is cloud diameter, D_m is maximum cloud diameter in an ensemble of clouds (population) and d is minimum diameter in this population, K is normalization coefficient and α – parameter, which depends on the time of day and various local climatic conditions.

Thus, the total number density of the cumuli, of the clouds of all diameter sizes in the population, is given by

$$n_t = \int_d^{D_m} n\, dD = \frac{K}{\alpha}\left(e^{-\alpha d} - e^{-\alpha D_m}\right).$$

Planck's model for cloudfields

And the number distribution equation is

$$n_f = K \int_{D-\epsilon/2}^{D+\epsilon/2} e^{-\alpha D}\, dD = K \delta_n e^{-\alpha D},\ d \leq D \leq D_m, \tag{11}$$

where

$$\delta_n = \frac{2}{\alpha} \sinh \frac{\alpha \epsilon}{2}.$$

The relation between the cloud power and its diameter is supposed to be

$$H = \eta D \left(\frac{D}{D_m} \right)^{\beta}, \tag{12}$$

where η and β are dimensionless parameters.

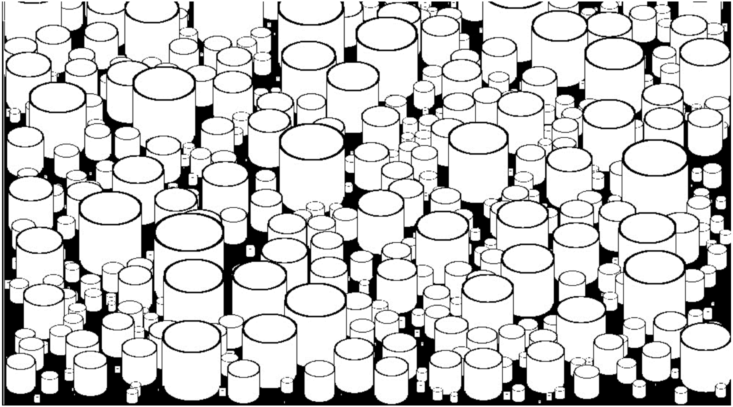


Figure 7: An example of Plank's cloudfield 3D

TABLE 2. Observed and analytically determined values of population parameters.

Sample Date	Time (EST)	Cloud base altitude h_b (ft)	Minimum cloud diameter d (ft)	Maximum cloud diameter D_m (mi)	Total number of cumuli N_T (no. per 100 mi²)	Population sky cover S_T	Total cloud volume V_T (mi³ per 100 mi²)	Maximum group diameter G_m (mi)	
10 Aug. 1957	0822	650	50	0.33	1480	0.084	0.62	None	
	0906	1800	50	0.70	1044	0.208	2.88	1.2	
	0945	2200	50	1.30	858	0.280	9.13	3.2	
	1055	3000	75	1.70	770	0.215	9.38	3.2	
16 Aug. 1957	0957	2550	50	0.85	901	0.096	1.35	1.6	
	1030	3000	50	0.70	1530	0.201	2.44	1.6	
	1138	3600	75	0.65	1562	0.173	2.53	1.6	
Typical sky cover	08–09	2200	50	0.50	1645	0.062	0.58	0.9	
	09–10	2300	50	0.70	1665	0.180	2.76	0.9	
	10–11	2700	50	1.30	1056	0.262	8.25	1.7	
	11–12	3000	75	1.30	1226	0.309	7.94	3.0	
	12–13	3650	75	1.60	1138	0.349	12.06	2.6	
	13–14	3500	100	2.10	513	0.477	26.76	4.2	
	14–15	4100	150	2.40	668	0.309	26.40	5.2	
	15–16	4500	200	2.50	609	0.185	10.02	3.1	
	16–17	4600	150	1.65	439	0.072	3.28	2.2	
Large sky cover	09–10	1800	50	0.90	2002	0.421	9.16	2.4	
	12–13	4000	75	2.50	1724	0.642	40.29	4.5	**no. 2**
	15–16	4500	100	2.50	1720	0.290	16.21	3.2	

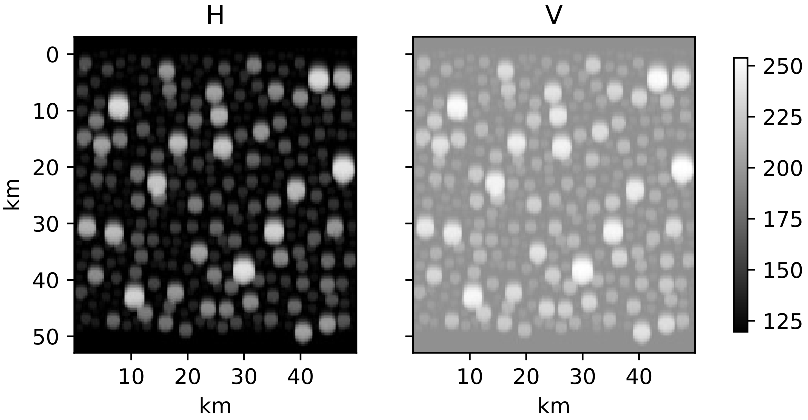
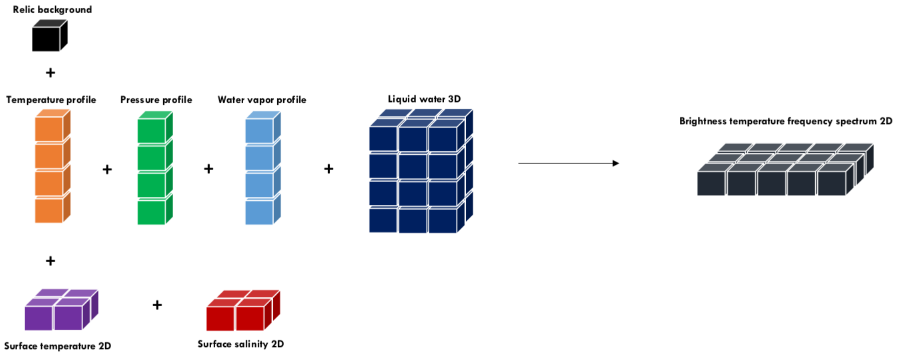
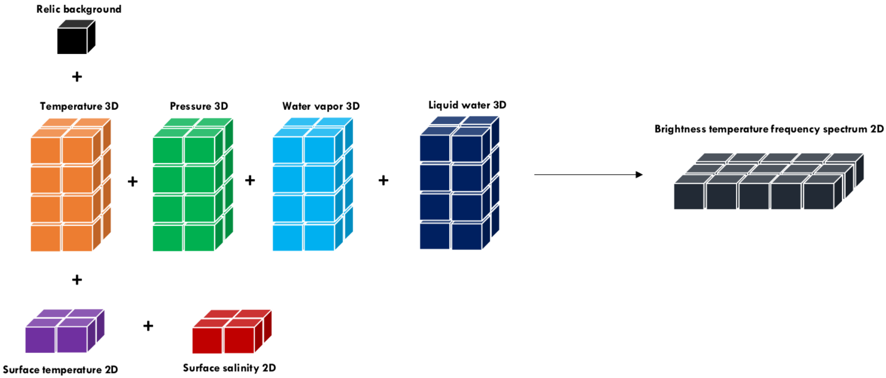


Figure 8: Simulation of brightness temperature 2D-map (51 deg. observation angle, 36 GHz frequency) for Planck's distribution no.2 "Large sky cover" over the standard atmosphere. Both horizontal (H) and vertical (V) polarizations are shown. Mazin's model is utilized for liquid water profile

Data flow [option 3]



Data flow [option 4]



Atmospheric moisture content parameters retrieval

Moisture content parameters are TVW (total water vapor)

$$Q = \int\limits_0^\infty \rho(h)\, dh$$

and LWC (liquid water content)

$$W = \int\limits_0^\infty \mathrm{w}(h)\, dh.$$

The dual-frequency method: if the total atmospheric opacity is retrieved and it is relatively small ($\tau \lesssim 1$ np), then it is sufficient to solve a system (13) of two linear equations written down for two different frequencies ν_1, ν_2 by any available method to get TVW and LWC

$$\tau_{\nu_i} = \tau_{O2}(\nu_i) + k_\rho(\nu_i) \cdot Q + k_w(\nu_i, t_w) \cdot W, \quad i = 1, 2, \tag{13}$$

Dual-frequency method

$$\tau_{\nu_i} = \tau_{O2}(\nu_i) + k_\rho(\nu_i) \cdot Q + k_w(\nu_i, t_w) \cdot W, \quad i = 1, 2,$$

where τ_ν is the retrieved total zenith opacity (np), $\tau_O(\nu)$ is model value of zenith opacity in oxygen (np), $k_w(\nu, t_w)$ is model weight function for attenuation in clouds

$$k_w(\nu, t_w) = \frac{60\pi \cdot \nu}{c} \mathrm{Im}\left(\frac{\varepsilon(\nu, t_w) - 1}{\varepsilon(\nu, t_w) + 2}\right), \tag{14}$$

t_w is an estimation of effective cloud temperature,
and weight function for water vapor $k_\rho(\nu)$ can be written, e.g. as

$$k_\rho(\nu) = \left(\int_0^\infty \gamma_{H2O}(\nu, h)\, dh\right) \cdot \left(\int_0^\infty \rho_{\mathrm{std}}(h)\, dh\right)^{-1}. \tag{15}$$

Let us consider approximations for brightness temperatures of upward $T_b^{\uparrow}(\nu,\theta)$ and downwelling $T_b^{\downarrow}(\nu,\theta)$ radiation

$$T_b^{\uparrow}(\nu,\theta) = T_{av}^{\uparrow}\left(1 - e^{\tau_\nu \sec\theta}\right), \quad T_b^{\downarrow}(\nu,\theta) = T_{av}^{\downarrow}\left(1 - e^{\tau_\nu \sec\theta}\right), \tag{16}$$

where $T_{av}^{\uparrow}$ and $T_{av}^{\downarrow}$ are average effective temperatures for upward and downwelling radiation respectively, $0 \le \theta \le 0.4\pi$.
Using these approximations, one can rewrite brightness temperature of outgoing radiation (1) as (17)

$$\begin{aligned} T^*_{\mathrm{v,h}}(\theta) = T_{av}^{\uparrow}\left[1 - e^{-\tau_\nu(\theta)}\right] + T_s \cdot \varkappa_{\mathrm{v,h}}(\theta) \cdot e^{-\tau_\nu(\theta)} + \\ + T_{av}^{\downarrow}\left[1 - e^{-\tau_\nu(\theta)}\right] R_{\mathrm{v,h}}(\theta) e^{-\tau_\nu(\theta)} + T_C \cdot R_{\mathrm{v,h}}(\theta) e^{-2\tau_\nu(\theta)}. \end{aligned} \tag{17}$$

Here $\tau_\nu(\theta)$ is understood as $\tau_\nu \cdot \sec\theta$.

The previous equation (17) is quadratic with respect to $e^{-\tau_\nu(\theta)}$. One can solve it for fixed polarization and thus get an estimation on total zenith opacity (18)

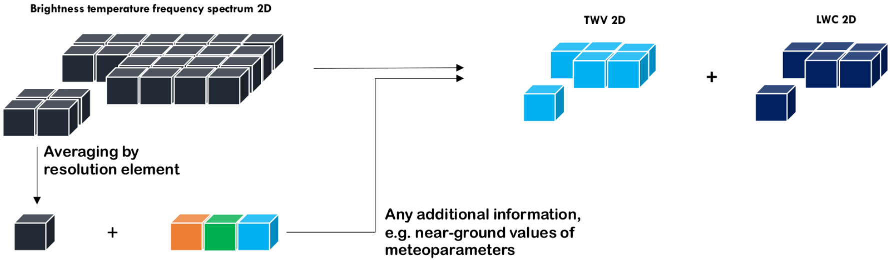
$$e^{-\tau_\nu(\theta)} = \frac{-b+\sqrt{D}}{2\cdot a} \quad \text{or} \quad \tau_\nu = \ln\left(\frac{2\cdot a}{-b+\sqrt{D}}\right)\cdot\cos\theta, \tag{18}$$

where

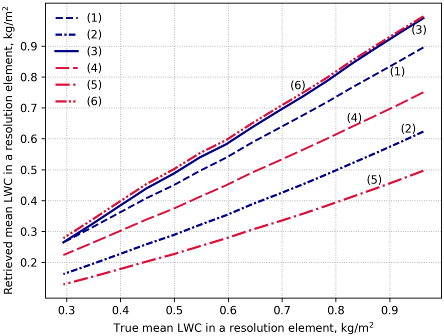
$$\begin{aligned} a &= \left(T_{av}^{\downarrow} + T_C\right)\cdot R(\theta), \\ b &= T_{av}^{\uparrow} - T_{av}^{\downarrow}\cdot R(\theta) - T_s\cdot\varkappa(\theta), \\ D &= b^2 - 4\cdot a\cdot\left(T^*(\theta) - T_{av}^{\uparrow}\right). \end{aligned}$$

Here $T^*(\theta)$ is the measured (registered by satellite radiometer) value of brightness temperature at ν-th frequency outgoing to θ-th direction.

Moisture content retrieval in 2D



Errors related to resolution element

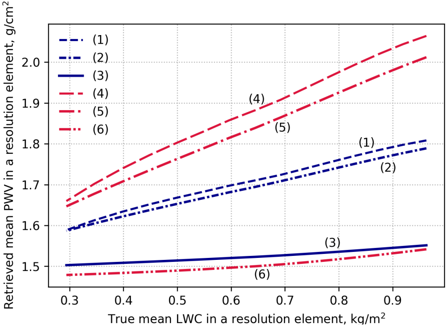


Blue: 22.2 and 27.2 GHz frequency pair.
Red: 22.2 and 36 GHz frequency pair.

(1) and (4) – first solving the inverse problem, then averaging LWC; (2) and (5) – first averaging BTs, then solving the inverse problem; (3) and (6) – first obtaining BTs of LWC-equivalent flat layer, then solving the inverse problem.

Figure 9: LWC dual-frequency retrieval from brightness temperatures (BTs) of "smooth water surface – atmosphere" system with added broken cloudiness (modified "Large sky cover" no.2) of various cover percentage (20–70%). Total area of population is 100×100 km. The size of radiometer antenna's resolution element is 10×10 km.

Errors related to resolution element



Blue: 22.2 and 27.2 GHz frequency pair.
Red: 22.2 and 36 GHz frequency pair.

(1) and (4) – first solving the inverse problem, then averaging TWV; (2) and (5) – first averaging BTs, then solving the inverse problem; (3) and (6) – first obtaining BTs of LWC-equivalent flat layer, then solving the inverse problem and getting TWV.

Figure 10: TWV dual-frequency retrieval from brightness temperatures (BTs) of "smooth water surface – atmosphere" system with added broken cloudiness (modified "Large sky cover" no.2) of various cover percentage (20–70%). Total area of population is 100×100 km. The size of radiometer antenna's resolution element is 10×10 km.

Conclusions

- A software framework for effective simulating the downwelling, upwelling and outgoing brightness temperature hyper-spectra from "smooth water surface – cloudy atmosphere" system geophysical parameters' distributions has been developed and tested (GPU).
- The inverse problem of atmospheric moisture content parameters' 2D-distributions dual-frequency retrieval has been solved.
- Planck's model for generating populations of cumuli has been considered. TWV and LWC retrieval errors related to the usage of flat–layered cloudfield approximation inside the radiometer antenna's resolution element (when solving the inverse problem) have been studied.

Thanks for your attention!