Parallel efficiency of monolithic and fixed-strain solution strategies for poroelasticity problems

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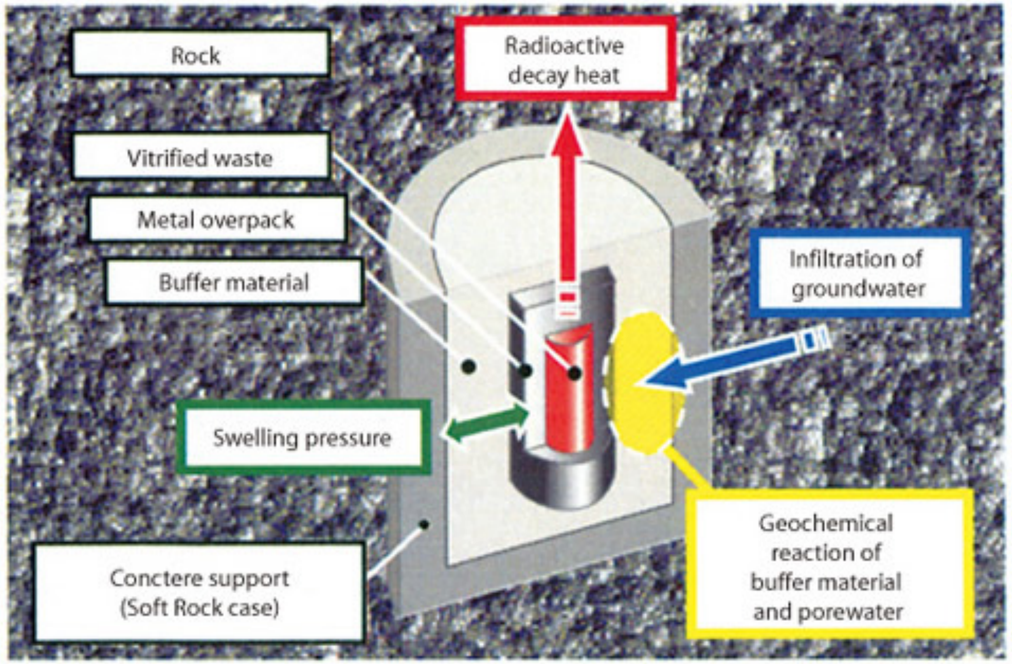
ИБРАЭ

Motivation



- Thermo-hydro-mechanical processes near radioactive waste repositories
- Coupled multyphysical problems
- Efficient parallel solvers are a must
- Poroelasticity is a simple first step
- **Scalability of different concepts** can be tested

GeRa (Geomigration of Radionuclides) – subsurface simulator

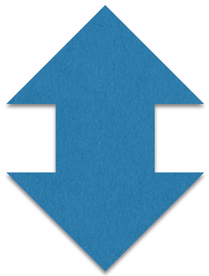
gera.ibrae.ac.ru

Poroelasticity

Poroelasticity

=

Flow in porous media



Elastic deformation of the media

Governing equations: Biot model

- Groundwater flow: mass conservation + Darcy's law + volume change

$$s_{stor}\frac{\partial h}{\partial t}-\nabla\cdot\left(\mathrm{K}\nabla h\right)+\alpha\nabla\cdot\frac{\partial \mathbf{u}}{\partial t}=Q$$

- Elasticity: mechanical equilibrium + Hooke's law + water pressure

$$\nabla\cdot\left(\mathbb{C}\frac{\left(\nabla\mathbf{u}\right)+\left(\nabla\mathbf{u}\right)^{\mathrm{T}}}{2}-\alpha P\mathrm{I}\right)=\mathbf{f}$$

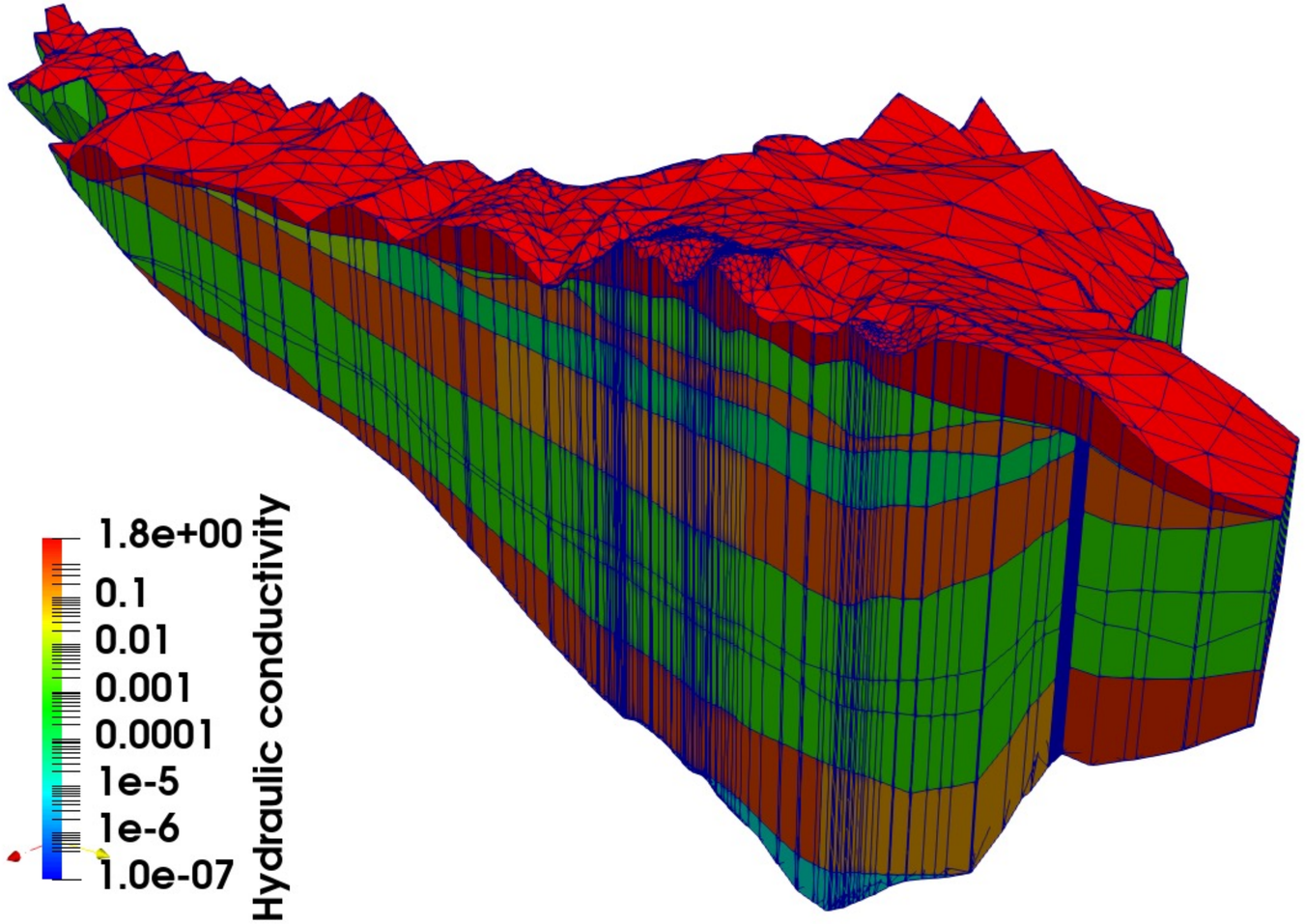
- Primary variables are water head h and solid displacement $\mathbf{u}$

Numerical solution: challenges

Unstructured grids:

- Layered domains
- “Flat” cells
- Cells can be general polyhedra

- Strong heterogeneity
- Anisotropy

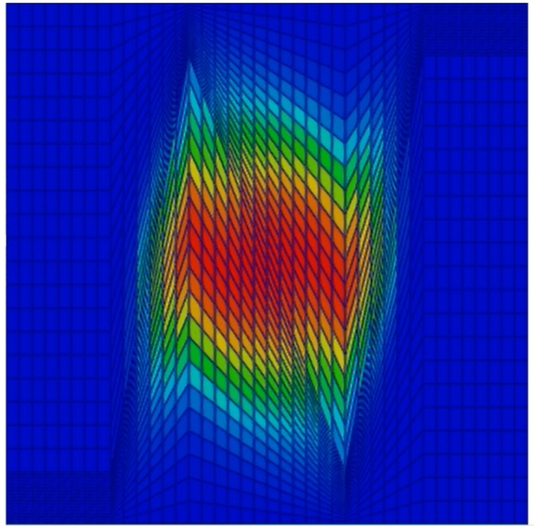


Spatial discretization: flow

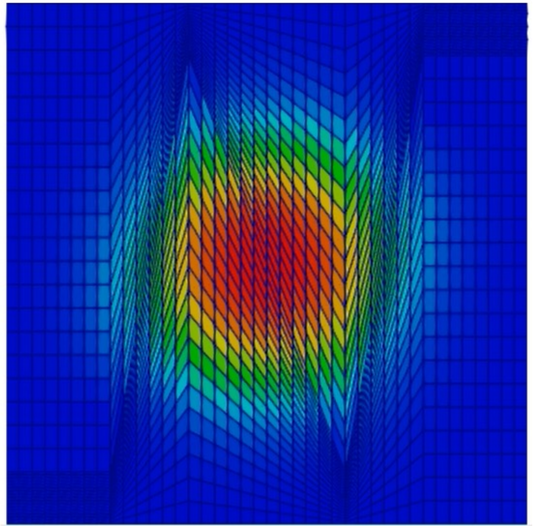
The finite volume method (FVM):

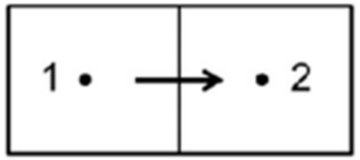
- Locally conservative
- Can handle wide class of cell shapes
- Easy to implement and is widely used
- Flux approximation is the key issue

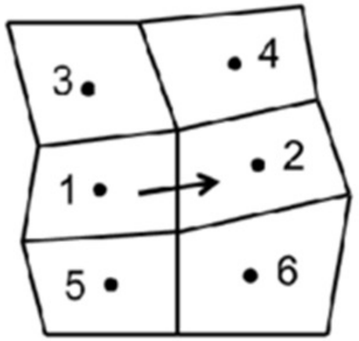
TPFA



MPFA-O



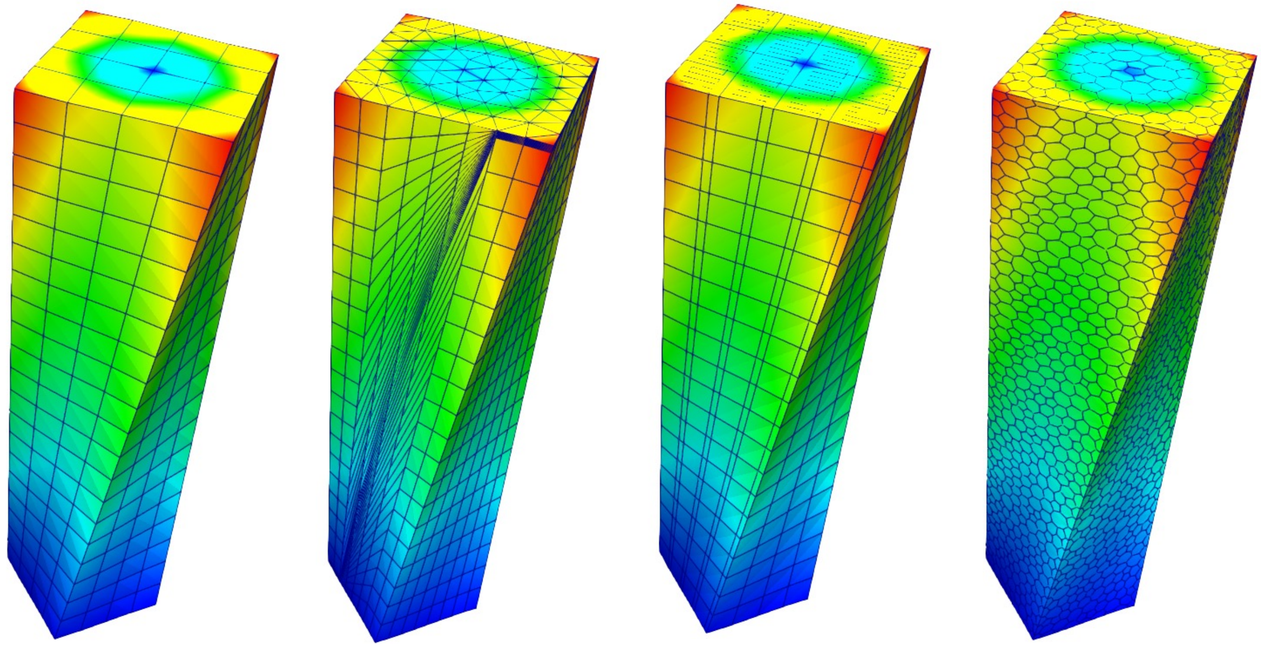


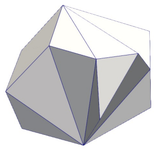


Spatial discretization: mechanics

Recently introduced virtual element method (VEM):

- Works on arbitrary cells
- Is algorithmically similar to conventional FEM
- Grows in popularity, gains theory
- Is used in multiphysics with FVM!





Temporal discretization

Fully implicit (backward Euler) scheme:

- Conventional for subsurface modeling
- Unconditionally stable
- Produces a linear system

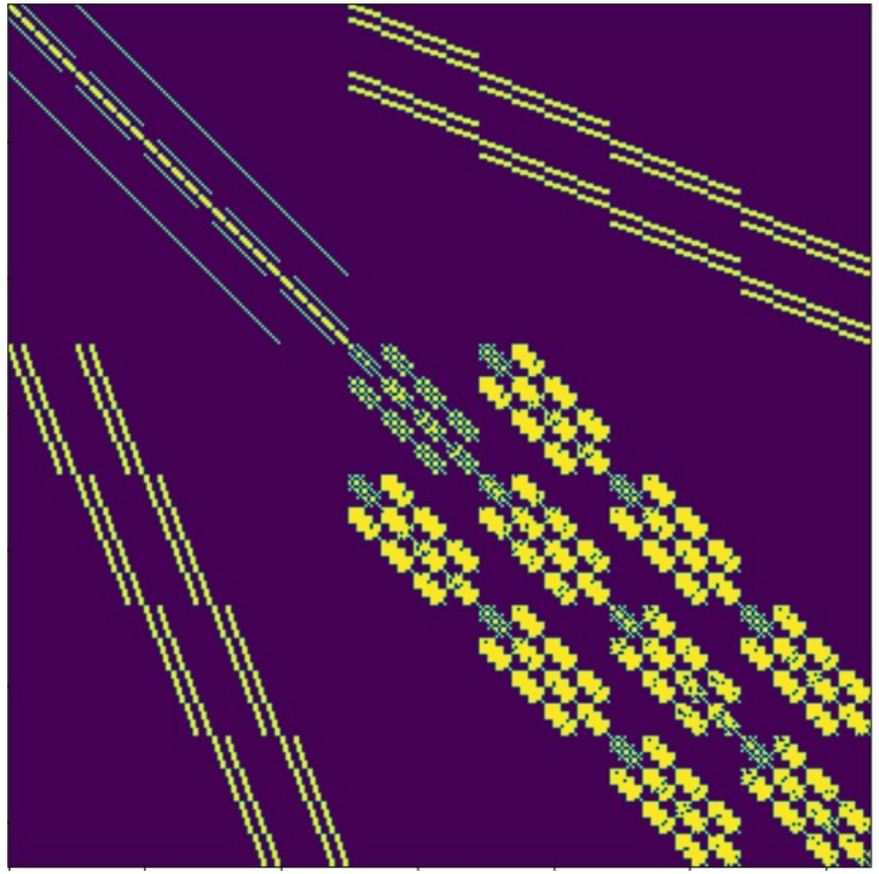
Structure of the coupled system

FVM for flow

VEM for the coupling terms

$$\begin{bmatrix} A_f & A_{fm} \\ A_{mf} & A_m \end{bmatrix} \cdot \begin{bmatrix} h \\ \mathbf{u} \end{bmatrix} = \begin{bmatrix} b_f \\ b_m \end{bmatrix}$$

VEM for mechanics



Solution strategies

Coupling

Monolithic

Solving the full system

- Unconditionally stable
- Large matrix
- Complicated matrix pattern

Fixed-strain

Sequential flow and mechanics substeps

- Can use tailored solvers
- Less memory-consuming
- Conditionally stable
- Adds an iterative loop on each time step

Implementation

The INMOST (www.inmost.org) numerical platform written in C++ provides

- Unstructured mesh handling
- Automatic differentiation tools for systems assembly
- Linear solvers
- MPI parallelization:
  - Mesh partitioning
  - Parallel linear solvers

Parallel efficiency test

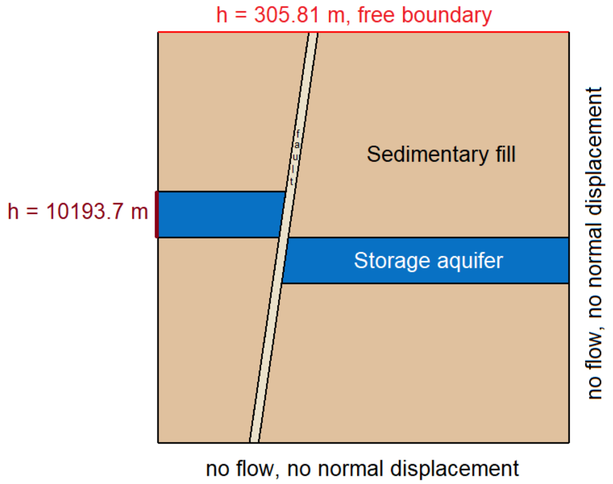
The idea:

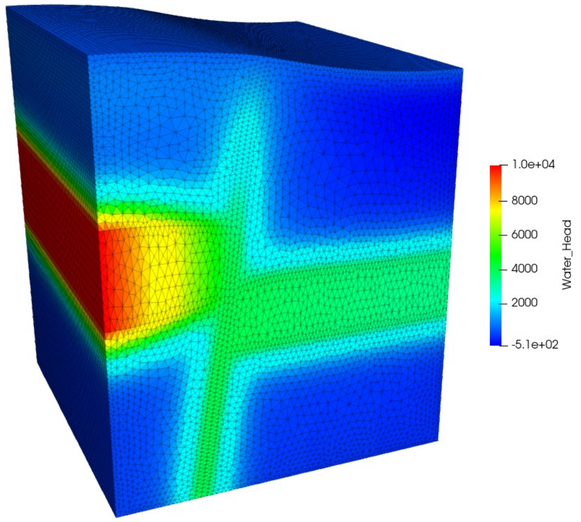
- Fixed-strain strategy solves **smaller** systems with **simpler structure**
- A general-purpose black-box linear solver with no tuning can work better
- INMOST solver `Inner_MPTILUC` was used

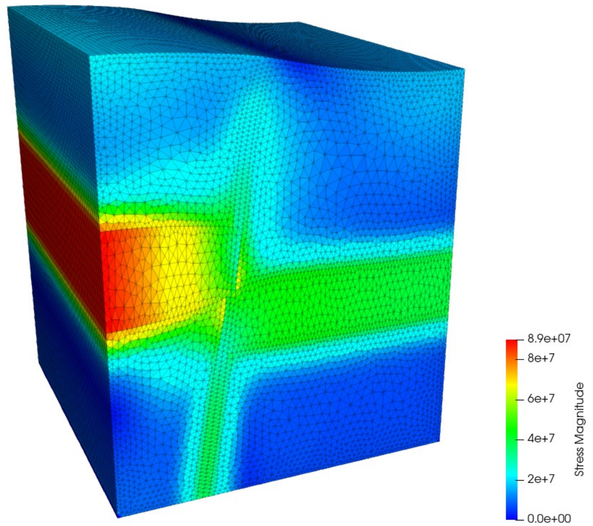
Problem A: faulted reservoir

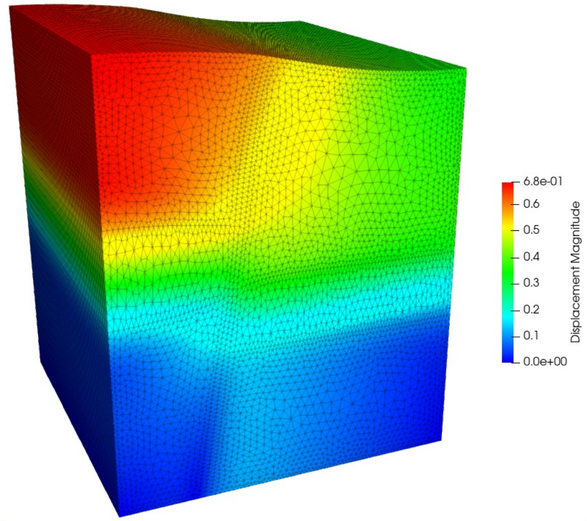
A 3-layer domain with fault

- 1 700 000 unknowns
- 4 time steps, 127 years
- 8-100 cores





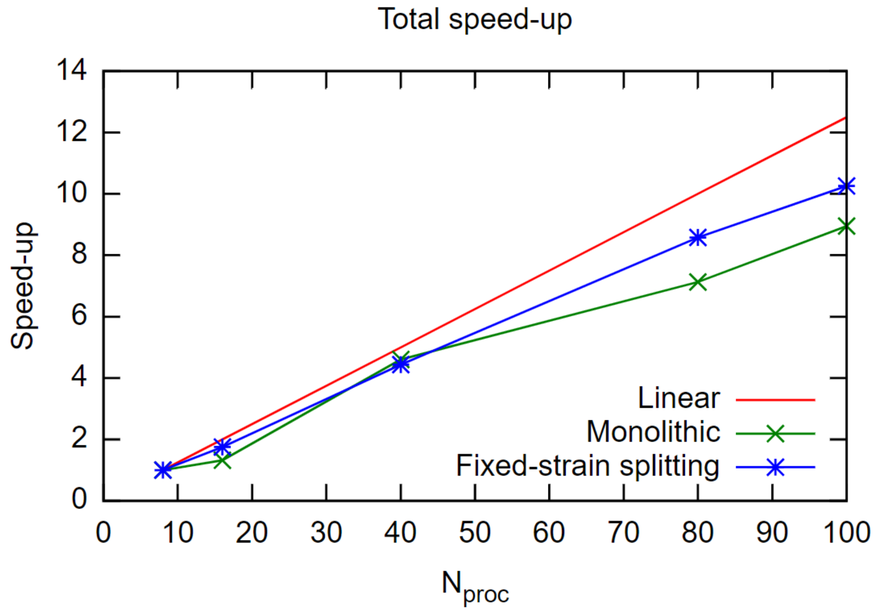


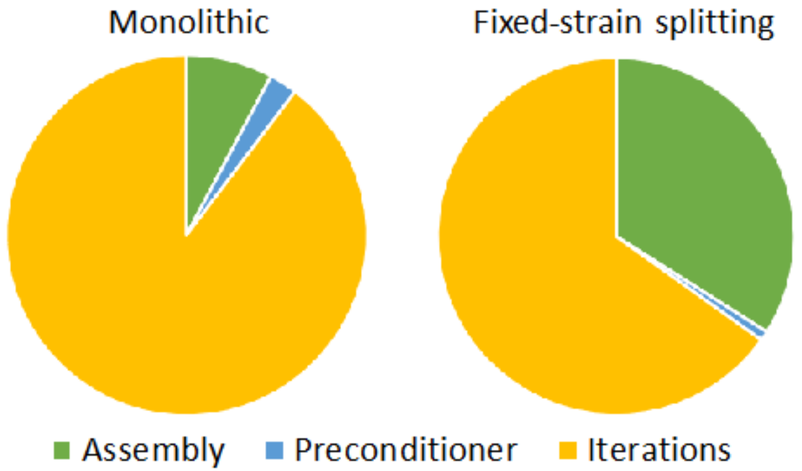


Problem A: results

Fixed-strain scales better

- Assembly takes larger fraction of time
- Assembly naturally scales better



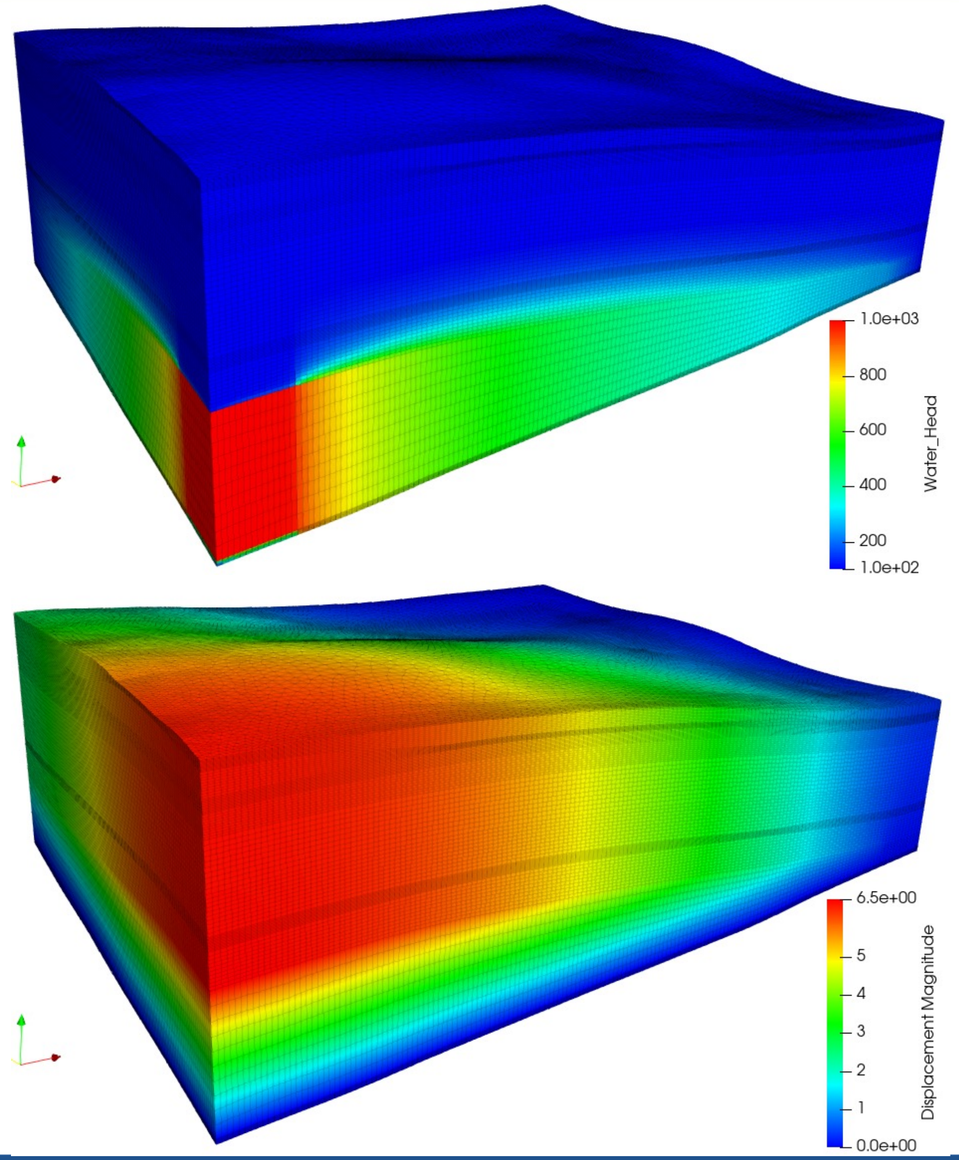


Problem B: real-life domain

A 9-layer domain, 11 media

Injection in 8th layer

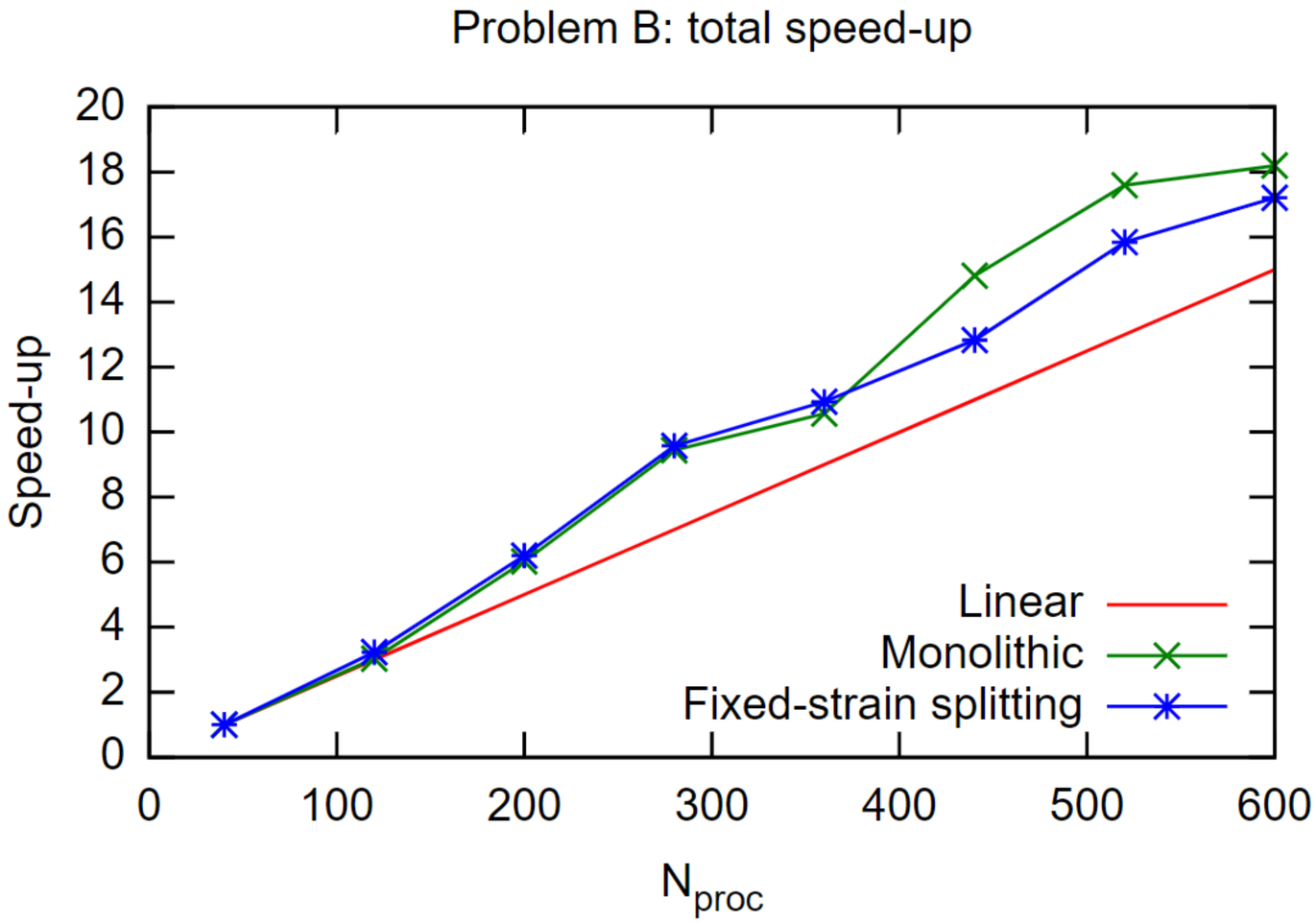
- 5 460 000 unknowns
- 2 time steps, 6 years
- 40-600 cores



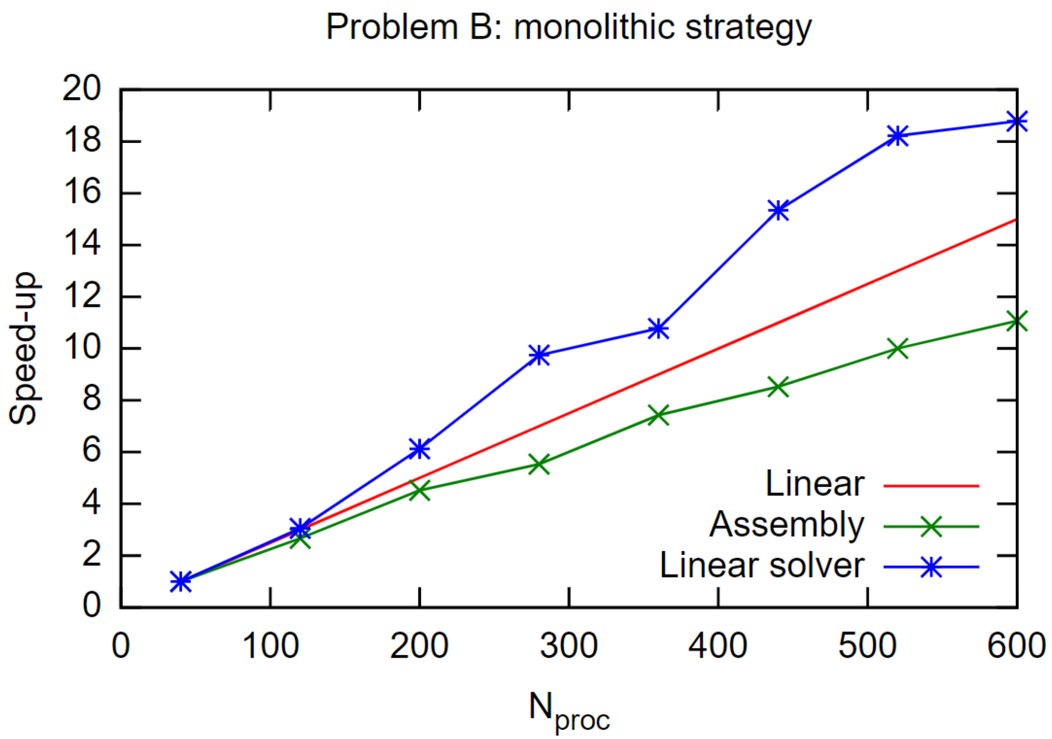
Problem B: results

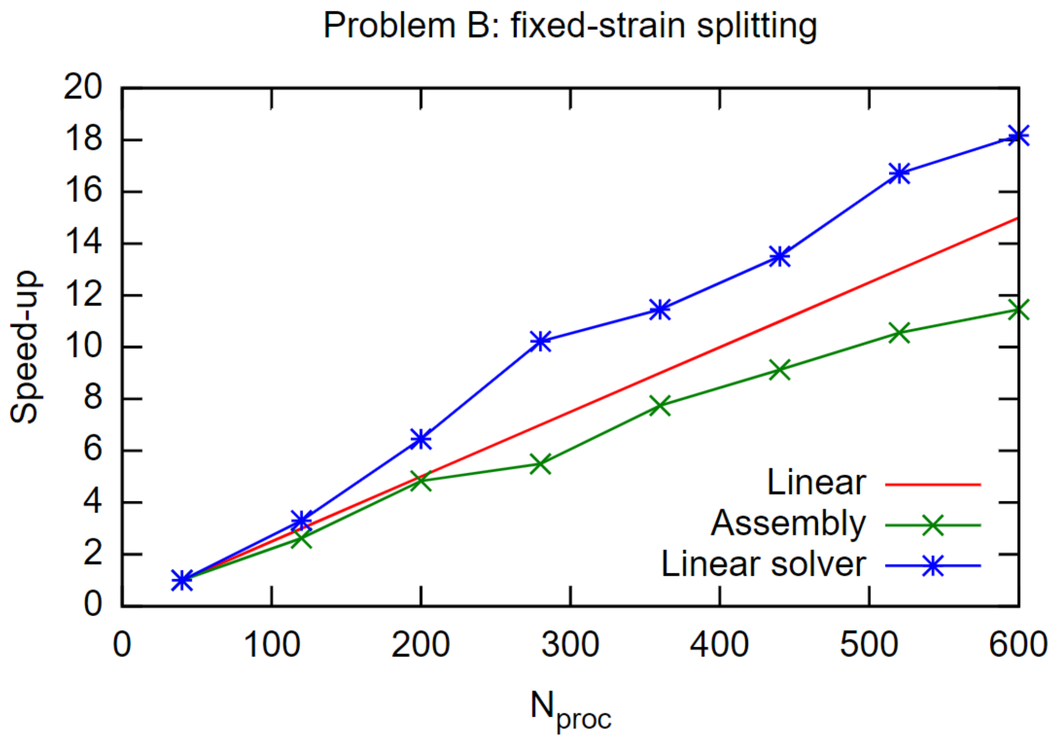
Both scale *superlinearly!*

Monolithic even scales better



Problem B: why superlinear?

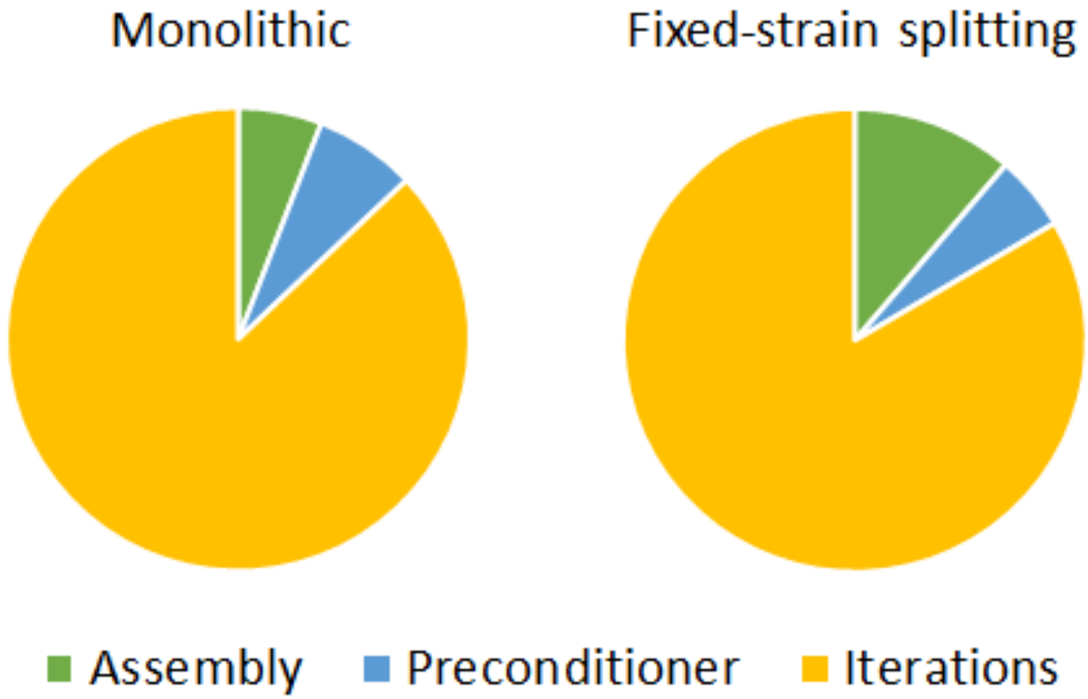




The reason is superior scaling of MPTILUC preconditioner, default drop tolerance makes it closer to full *LU*-decomposition

Problem B: why sublinear assembly scaling?

- Non ideal mesh partitioning
- Assembly takes larger fraction of time in fixed-strain strategy
- It’s the reason why fixed-strain scales worse



Conclusions

- Efficient solvers are required for multyphics
- Monolithic and splitting strategies are considered for poroelasticity problems discretized on unstructured meshes
- Strategies were tested in parallel with **no tuning** of linear solver or mesh partitioner
- No clear answer on which scales better
- Side note: INMOST linear solvers can handle coupled systems

Thank you for your attention!